Making Means: Data Distribution and Averages

For GSP5 ACTIVITY NOTES

INTRODUCE

Project the sketch for viewing by the class. Expect to spend about 5 minutes.

1. Open **Making Means.gsp** and go to page "Line Plot."
2. Explain, ***Today you're going to use Sketchpad to create data sets that have specific mean values. Usually, you start with a given data set and have to calculate the mean. But now you'll work in the reverse direction. If you're given a certain mean value, can you think of how many data sets it might represent? How big would the data set have to be? How might the different values in the data set be distributed? Before you begin, I'll demonstrate how to create a data set using the sketch.***
3. Drag the data point from 7 to 6. Students should see that the mean changes as you do this. Drag a new data point to the number line and ask the students where to place the data point in order to make the mean smaller, bigger, and the same.
   - Show that data points can be stacked.
   - Show students that dragging a data point back to the left bank removes it from the set.
   - Press *Reset* to return to the initial configuration of the sketch.

DEVELOP

Expect students at computers to spend about 25 minutes.

4. Assign students to computers and tell them where to find **Making Means.gsp.** Distribute the worksheet. Tell students to work through step 16, and do the Explore More if they have time. You might consider convening the class after most students have completed worksheet step 11 and discussing students' predictions before asking students to continue on with the remaining questions.
5. Let pairs work at their own pace. As you circulate, here are some things to notice.
   - For questions where students are asked to provide more than one example, encourage them to construct examples that are different. For instance, {3, 4, 5}, {2, 4, 6}, and {1, 4, 7} are similar examples of data sets with means of 4.00. However, {0, 6, 6} is quite different because it neither includes 4 nor has distinct values.
   - Encouraging students to see what their sets of data have in common might help them construct different data sets. (Sometimes it's hard for

students to think outside a particular strategy they have developed to generate new data sets).

- For worksheet step 10, encourage students to write down things they know for sure and things they don't know for sure. Sometimes the things you don't know for sure can be just as important!
- In talking about the prediction for worksheet step 11, ask students to explain their predictions. Students can either use examples or describe how the sum of the numbers and total number of data points will change when adding a new data point.

6. If you decide to convene the class once most students have completed worksheet step 11, follow steps 7–10 in the Summarize section before having pairs return to their computers.

SUMMARIZE

Project the sketch. Expect to spend about 15 minutes.

7. Gather the class. Students should have their worksheets with them. Open **Making Means.gsp** and use it to support the class discussion.
8. Ask students to offer solutions to worksheet step 4, and write them down. After you have written down several, ask students for examples that are different from the ones written on the board. Help the students notice that not all data sets with a mean of 5 will be "balanced" around the mean or will include the value 5.
9. Now discuss the question in worksheet step 9, and ask students to talk about their strategies. Some will describe how they constructed their data sets so that the values always summed to 20. Some might also have considered the data points in pairs, and thought about summing a given pair to 10. If students are having difficulty using the sum to help them generate new data points, illustrate how they might do this using fewer data points and a different mean.
10. Students have now worked quite a bit on constructing different sets of data. Using the sketch, drag five data points to these locations: 2, 3, 4, 5, and 6. ***How does the mean change as more data points are added? Can the mean stay the same? Can it increase? Decrease? How do you know?*** Ask for volunteers to explain their reasoning. Then drag a new data point to 4, and drag it slowly toward the right so that the students see the mean value continuously increasing. Return it to 4, and ask a

student to describe what will happen if the value moves toward the left. This should help students see that the mean must change unless the new data point has the same value as the previous mean.

11. Continue on to worksheet step 14. Ask for volunteers to describe their answers. Some will have found a way of getting a mean of 10 by dragging the square off the screen. For those who stayed within the bounds, ask them to explain why the mean could never be 10. They may simply want to show you that they placed all of the remaining data points on 10, and still the mean was less than 10. They should be able to explain that at least one data value would have to be greater than 10 for the mean to be 10.00 given the existing data points. To provide some extra challenge: ***What if we placed three of the four remaining data points at 10? Then what value would the final data point have to be in order to make a mean of 10?*** [46]

12. For worksheet step 15, ask students to share some of the data points they had. Again, make sure to ask for different data sets. ***So, given these data sets, what can we say about a family of 8 that has a mean height of 4 feet?*** Encourage students to take into account the heights that the family members could realistically have.

EXTEND

1. ***When do you think that the mean would be a good measure to use to describe a set of data?*** Encourage students to give specific examples. They may have already encountered the mean in the context of school grades. Usually, what's important is not so much the kind of data that is being collected (grades), but what we want to know about the data. So, if we want to know whether a test was too hard for most students, then the mean is helpful because it is sensitive to extreme values (in this case, to very low test scores). However, the mean might not be a very good measure if we don't care about extreme values (maybe some students were away, so we don't want their marks of 0 to affect the measure).

2. Have pairs work together with the sketch. Introduce the idea of the median, if the students haven't already encountered it. Ask the students to find a few examples where the mean will equal the median for a given data set.

ANSWERS

3. Answers will vary. Sample answers: {3, 4, 6, 7}, {4, 4, 5, 7}, or {2, 6, 6, 6}
4. Answers will vary. See above.
6. {0, 9, 9} or {0, 8, 10}. Other answers are possible if data points beyond 10 are used.
9. Answers will vary. Sample answer: To keep a given mean when adding two data values, balance the two data points around the mean.
10. The data points can be distributed in many ways, and can even include extreme values such as 0 and 10. None of the data points has to be 4.00. However, the mean of all the data points must equal 4.00.
11. Answers will vary.
12. If the new data value is greater than 4, then the mean will increase. The larger its value, the more the mean will increase. Similarly, if it's less than 4, the mean will decrease, with smaller values decreasing it more.
14. Only if some of the four new data points have values more than 10. For the ten data values to have a mean of 10, the sum of the ten data points would have to be 100, which is 76 more than the sum of the given data values.
15. Answers will vary. The sum of the heights must be 32.
16. Because the question involves the heights of humans, it is unlikely that any person will be more than 7 feet tall and unlikely that any person will be less than 1 foot tall. In order to get a mean of 4, we know that either all of the heights are exactly 4, or at least some will be less than 4 feet tall, and some will be more.
17. The sum of the three points is 27, and the sum of the first two is 10, so the third point is 17.