Meet the Parallelogram: Properties of Parallelograms

For GSP5 ACTIVITY NOTES

INTRODUCE

Project the sketch for viewing by the class. Expect to spend about 10 minutes.

1. Open Sketchpad and enlarge the document window so it fills most of the screen.
2. Explain, ***Today you're going to use Sketchpad to construct a parallelogram and investigate its properties. First I'll demonstrate how to construct a parallelogram using its definition. As I construct the parallelogram, think about how you would write the definition.*** As you demonstrate, make lines thick and labels large for visibility. Model the parallelogram construction in worksheet steps 1–12. Here are some tips.
   - In worksheet steps 1–5, you will construct one side of the parallelogram and a line parallel to it. ***What does it mean for two lines to be parallel?*** [The lines never intersect.] ***What does $\overline{AB}$ represent?*** [One side of the parallelogram] ***What do you think point C is?*** [A vertex]
   - In worksheet step 6, you will construct a second side that is adjacent to $\overline{AB}$. ***What does $\overline{AC}$ represent?*** [Another side of the parallelogram] ***What is its relationship to $\overline{AB}$?*** [It is next to, or adjacent to, it.]
   - In worksheet steps 8 and 9, model how to use the **Point** tool to construct the point of intersection (although you can also construct an intersection point by clicking the intersection with the **Arrow** tool). Explain that both lines will be highlighted when the **Point** tool is in the right place to construct the point of intersection. ***What is point D?*** [A vertex]
   - In worksheet steps 10 and 11, explain that two sides of the parallelogram are lines, so you will hide them and construct segments instead. Ask students to name the finished parallelogram. [Parallelogram *ABDC* or *ACDB*] Review that parallelograms are named by their vertices. Depending on your curriculum, you might introduce the symbol for parallelogram: $\parallelogram$.
   - In worksheet step 12, demonstrate how to drag the different parts of the parallelogram to test the construction. Explain that this is an important step; if the parallelogram is not constructed properly, students can choose **Edit | Undo** as needed.

3. ***How does this construction help you define* parallelogram*?*** Students may make the following response.

   Based on the construction, we know side AB is parallel to side CD and that side AC is parallel to side BD. The sides are not adjacent, but are opposite. So a definition would be that opposite sides are parallel.

4. Encourage students to add to the definition if it is not complete. ***How many sides does a parallelogram have?*** [Four sides] ***What type of polygon is it?*** [Quadrilateral]

5. After students have come up with a complete definition, write it on chart paper: *A parallelogram is a quadrilateral whose opposite sides are parallel.*

6. If you want students to save their work, demonstrate choosing **File | Save As,** and let them know how to name and where to save their files.

DEVELOP

Expect students at computers to spend about 25 minutes.

7. Assign students to computers. Distribute the worksheet. Tell students to work through step 21 and do the Explore More if they have time. Encourage students to ask their neighbors for help if they are having difficulty with the construction.

8. Let pairs work at their own pace. As you circulate, here are some things to notice.

   - In worksheet step 13, ask students what they observe about the lengths of the sides. Students should notice that opposite sides are congruent. ***Do you think this holds true for any parallelogram?*** Have students think about this before dragging the parallelogram and making their conjectures.

   - In worksheet step 14, be sure students choose the vertex as the second point when measuring an angle. Also, check that students don't measure the same angle twice. Encourage students to look for relationships among the angle measures. ***What do you notice about angles opposite each other? How about the consecutive angles, the angles next to each other?*** Students should observe that opposite angles have the same measure and adjacent angles are supplementary.

Review the term *supplementary,* if needed. ***What are supplementary angles?*** [Angles whose measures add to 180°.] ***Can you find any of these types of angles?***

- In worksheet step 15, as students drag parts of the parallelogram, encourage them to make different types of parallelograms. ***Now make another parallelogram that looks completely different. What changes as you drag parts of the parallelogram? What stays the same?***
- In worksheet step 17, check that students understand that a diagonal is a line segment that connects two nonadjacent vertices. ***How will you construct the diagonals of the parallelogram?*** [Construct $\overline{AD}$ and $\overline{CB}$]
- In worksheet step 20, students will be measuring distances between points, although they could also construct new segments from the vertices to the point of intersection and then measure their lengths. Again, remind students to drag the parts of the parallelogram to form many different parallelograms. ***What do you notice about the distances you measured?***
- If students have time for the Explore More, they will use the properties of a parallelogram to construct parallelograms using different methods. Be sure students clearly describe their construction methods and explain what properties they used. The more time you give students, the more methods they will find. Give students hints to start their thinking, if necessary. Have students drag vertices of their figures to make sure their constructions are correct. Parallelograms that fall apart and can turn into other shapes are underconstrained. A construction that stays a parallelogram but that can't take on all possible shapes of a parallelogram is overconstrained.

9. If students will save their work, remind them where to save it now.

SUMMARIZE

Project the sketch. Expect to spend about 10 minutes.

10. Gather the class. Open **Meet Parallelogram Present.gsp** and use pages "Sides and Angles" and "Diagonals" as needed. Students should have their worksheets with them. Begin the discussion by writing "Properties of Parallelograms" on chart paper. ***We started this activity by constructing a parallelogram based on its definition. What property did we use to construct the parallelogram?*** Write down the response

on the chart paper. Then review the conjectures students made about the sides, the angles, and diagonals of parallelograms. Volunteers may come to the computer and drag the parallelogram to show how the conjectures hold true for different parallelograms.

Properties of Parallelograms
Opposite sides are parallel.
Opposite sides are equal in length.
Opposite angles have equal measures.
Consecutive angles are supplementary.
Diagonals bisect each other.

11. Drag the parallelogram so that it is a rectangle, having four right angles. ***What shape does this look like?*** [Rectangle] ***Is it still a parallelogram? Explain.*** [Yes, it still has the properties of a parallelogram.]
12. Drag the parallelogram so that it is a rhombus, having four congruent sides. ***What shape does this look like?*** [Rhombus] ***Is it still a parallelogram? Explain.*** [Yes, it still has the properties of a parallelogram.]
13. Now drag the parallelogram so that it is a square, having four right angles and four congruent sides. It might be easier if you select one point at a time and use the arrow keys on your computer to move it in small increments. ***What shape does this look like?*** [Square] ***Is it still a parallelogram? Explain.*** [Yes, it still has the properties of a parallelogram.]
14. If time permits, discuss the Explore More. Have students review their construction methods and the properties they used in each. The last four pages of **Meet Parallelogram Present.gsp** show some methods that you may wish to share with students.
15. ***The measure of one angle of a parallelogram is 60 degrees. What are the measures of the other angles? Explain your reasoning.*** [Opposite angles are congruent, so the opposite angle measures 60°. Consecutive angles are supplementary, so the two other angles each measure 120°.]

EXTEND

1. Have students draw a Venn diagram showing the relationship between quadrilaterals, parallelograms, rectangles, rhombuses, and squares.

2. Have students explore what conditions are necessary for a quadrilateral to be a parallelogram. Ask students to construct a quadrilateral where the given condition is true and then state whether the quadrilateral is always, sometimes, or never a parallelogram.

Condition	Always, Sometimes, Never a Parallelogram
The diagonals bisect each other.	Always
The diagonals are congruent.	Sometimes (when the parallelogram is a square diagonals bisect each other, making a rectangle)
Opposite angles are 45° and 50°.	Never
Both pairs of opposite sides are congruent.	Always

3. Ask students to try to construct a parallelogram with all its vertices on the circumference of a circle. ***Were you able to construct a parallelogram? If so, what type of parallelogram did you construct?*** [Rectangle]

ANSWERS

16. Opposite sides in a parallelogram are equal in length. Opposite angles in a parallelogram are equal in measure. Consecutive angles in a parallelogram are supplementary.

21. Diagonals in a parallelogram bisect each other. Students might also notice that the diagonals in a parallelogram divide the parallelogram into four pairs of congruent triangles.

22. Students may find one or more of the following methods.

    Method: Construct a segment and a point not on the segment. Mark the endpoints of the segment as a vector. Translate the point not on the segment by the marked vector. Construct the missing sides.
    Properties: A parallelogram has one pair of opposite sides that are both parallel and congruent.

    Method: Construct a segment and a point not on the segment. Select the segment and the point and choose **Construct | Circle by Center+Radius**. Construct a line through the point, parallel to the segment. Find the point where the circle and the line intersect. This is the fourth vertex of the parallelogram.

Properties: A parallelogram has one pair of opposite sides that are both parallel and congruent.

Method: Construct a segment and its midpoint. Construct a line through the segment attached at the midpoint. Construct a circle centered at the midpoint. Find the two points where the circle intersects the line. These two points and the original endpoints of the segment are the vertices of the parallelogram.
Properties: The diagonals of a parallelogram bisect each other.

Method: Construct a pair of concentric circles. Construct two lines that both contain the center point. Find the two points of intersection of one of the lines with one of the circles and of the other line with the other circle. These four points are the points of intersection of the parallelogram.
Properties: The diagonals of a parallelogram bisect each other.