Parallel Pairs: Parallelogram and Triangle Area

For GSP5 ACTIVITY NOTES

INTRODUCE

Project the sketch for viewing by the class. Expect to spend about 10 minutes.

1. Open Sketchpad and enlarge the document window so it fills most of the screen.
2. Explain, ***Today you're going to use Sketchpad to find the areas of parallelograms and triangles and to discover how they are related. You'll use a process called*** **shearing** ***to help you write the area formulas. First I'll demonstrate how to construct a parallelogram and then how to shear it.***
3. As you demonstrate, make lines thick and labels large for visibility. First model the parallelogram construction in worksheet steps 1–12. Then model how to shear the parallelogram in worksheet step 14. Here are some tips.

   - Start off by discussing the properties of parallelograms. ***How would you define a parallelogram?*** Listen for the following points.

     It's a polygon with four sides; it's a quadrilateral.

     The opposite sides are parallel.

     The opposite sides are congruent.

     Write a definition for *parallelogram* on chart paper. Here is an example: A parallelogram is a quadrilateral whose opposite sides are parallel.

   - In worksheet step 1, model how to set the Sketchpad Preferences so that points are automatically labeled. ***For this sketch, we'll have Sketchpad automatically label the points as we construct them. It will be easier to follow the construction.***

Instead of using the **Segment** tool to make side $\overline{FG}$, it may be easier for some students to select the points F and G and then choose **Construct | Segment.**

   - In worksheet steps 2–5, model how to make two parallel lines. ***We'll use these two lines to help us construct two sides of the parallelogram. What do we need to do next?*** [Make another pair of parallel lines to construct the other two sides of the parallelogram.]
   - Follow worksheet steps 6–12 to demonstrate how to make the other pair of parallel sides. ***How does this figure meet our definition of a parallelogram?*** [It has four sides and the opposite sides are parallel.] If your curriculum's definition of a parallelogram includes that the opposite sides are congruent, you can measure the lengths of sides or the distances between vertices to verify that this condition is met.

- ***What is the name of this figure?*** [Parallelogram *DEGF*] Remind students that parallelograms are named by the vertices listed in order.
- In worksheet step 14, demonstrate how to shear the parallelogram by dragging point *E*. ***What happens when I shear the parallelogram?*** [The shape of the parallelogram changes.] ***Is the figure still a parallelogram no matter where I drag point E? Explain.*** [Yes, because opposite sides are still parallel.] Tell students that they will explore shearing a parallelogram more on their own.

4. If you want students to save their work, demonstrate choosing **File | Save As,** and let them know how to name and where to save their files.

DEVELOP

Expect students at computers to spend about 35 minutes.

5. Assign students to computers. If you decide to have students skip the parallelogram construction, tell them where to locate **Parallel Pairs.gsp.** Distribute the worksheet. Tell students to work through step 28 and do the Explore More if they have time. Encourage students to ask their neighbors for help if they are having difficulty with the construction.

6. Let pairs work at their own pace. As you circulate, here are some things to notice.
   - In worksheet step 14, make sure students understand that in this construction, a base of the parallelogram is $\overline{DF}$ or $\overline{EG}$ and the height is the shortest distance between a base and its opposite side—a perpendicular segment. Ask students to observe what happens to the base and its height when they drag each time. ***What happens to the base when you drag point E?*** [Nothing, it stays the same.] ***What happens to the height?*** [Nothing, it stays the same.] ***Now drag point D toward point F. What happens to the base?*** [It gets shorter.] ***What happens to the height?*** [Nothing, it stays the same.] ***Now drag*** $\overline{EG}$ ***down. What happens to the base?*** [Nothing, it stays the same.] ***What happens to the height?*** [It gets shorter.] Let students finish exploring on their own.
   - In worksheets steps 16 and 17, students construct a height for the parallelogram. ***How do you know that*** $\overline{EH}$ ***is the height?*** [It is perpendicular to the base $\overline{DF}$.] *Note:* $\overline{EH}$ is also perpendicular to opposite side $\overline{EG}$; students may not see this initially.

- In worksheet step 18, be sure students select only the perpendicular line and not $\overline{EH}$. The perpendicular segment, or height, should still remain after hiding the perpendicular line. Show students how to choose **Edit | Undo** if they make an error.
- In worksheet step 20, students measure $\overline{DF}$, a base of the parallelogram. ***Would you get the same result if you measured the distance between points E and G? Explain.*** [Yes, because opposite sides are congruent] If necessary, have students measure the distance between vertices. ***What is another name for*** $\overline{EG}$***?*** [It is also a base of the parallelogram.]
- In worksheet steps 21 and 22, listen to students as they derive the formula. ***Do you see a relationship between the base, the height, and the area? Go back and try dragging your parallelogram as described in worksheet step 14. What happens?***
- In worksheet step 25, ask students to define parts of $\triangle DEF$. ***What does*** $\overline{EF}$ ***represent?*** [A side of the triangle] ***What does*** $\overline{DF}$ ***represent?*** [A side and a base of the triangle] ***What does*** $\overline{EH}$ ***represent?*** [The height of the triangle] ***How do you know?*** [It is perpendicular to the base $\overline{DF}$.] Check that students understand that the triangle and the parallelogram share the same base and height.
- In worksheet step 27, question students as they drag point *E*. ***What happens to the base?*** [It stays the same.] ***To the height?*** [It stays the same.]
- If students have time for the Explore More, they will make an action button to animate point *E* along its line. The animation will shear the parallelogram.

7. If students will save their work, remind them where to save it now.

SUMMARIZE

Project the sketch. Expect to spend about 15 minutes.

8. Gather the class. Students should have their worksheets with them. Open **Parallel Pairs Present.gsp** and go to page "Parallelogram." Begin the discussion by having students identify the height and a base of the parallelogram. Write definitions for *height* and *base* on chart paper. Here are sample definitions: The height is the perpendicular distance from a base to the opposite side. The base is a side of the parallelogram. For your understanding, height is also referred to as "altitude."

9. Check students' understanding of these terms before proceeding. ***Is the base always the bottom of the parallelogram?*** [No, the base can be any side.] ***What is the height if $\overline{DE}$ is a base?*** [The perpendicular line segment from $\overline{DE}$ to the opposite side] ***Can the height lie outside of the parallelogram?*** [Yes] Demonstrate this concept by dragging points D or F to show the height outside of the figure.

10. Review worksheet step 15 with students. ***Did shearing the parallelogram change its area?*** [No] ***Why not?*** [The base and the height measurements stayed the same.] ***What measurements affect the area of the parallelogram?*** [The base and the height] ***Is this true for any size parallelogram?*** [Yes] Model this on the sketch.

11. Ask volunteers to share their formulas for area of a parallelogram. ***How did you arrive at your formula?*** Students' answers will vary. Here are some possible replies.

    The area changed when we changed the height and the base, so we looked for ways that the height and the base could equal the area.

    We noticed that the area is always greater than either the height or the base, so we tried adding the two. When that didn't work, we multiplied them.

    We noticed that you could make it into a rectangle, so we multiplied the base and the height.

    We changed the height and the base to whole number units so we could see the relationship more easily. When the height was 3 cm and the base was 4 cm, the area was 12 cm^2. We knew that $3 \times 4 = 12$, *so we thought the area might be height times base. We verified our guess by trying other measurements and it worked!*

    Use the Sketchpad Calculator to confirm that the product of the height and base is equal to the area. Write the formula on chart paper: $A = bh$.

12. Go to page "Triangle" and have students define the height and the base of $\triangle DEF$. ***What is the height of the triangle?*** [$\overline{EH}$] ***What is the base?*** [$\overline{DF}$] ***How do our definitions for height and base of a parallelogram compare to those for a triangle?*** [In both cases, the base is a side of the figure. The height of both figures is the perpendicular segment from a base; however, in a parallelogram, it is to the opposite side, and in a triangle, it is to the opposite vertex.] Write definitions for *height* and *base* of a triangle on chart paper: The height is the

perpendicular distance from a base to the opposite vertex (or the parallel line containing the opposite vertex). The base is a side of the triangle. Drag point D toward point F to model how the height is sometimes drawn outside the triangle.

13. Discuss worksheet steps 27 and 28. Drag point E in the example sketch. ***What happens to the area of the triangle when I drag point E?*** [The area stays the same.] ***Which measurements affect the area of the triangle?*** Drag points D or F or $\overline{DF}$. [The base and the height] ***How does this compare to what happens with a parallelogram?*** [The height and the base measurements affect the area of a parallelogram too.] ***What measurements do you need to find the area of a triangle?*** [The height and the base] ***How do you think the area of a triangle is related to the area of a parallelogram?*** Here are some possible responses.

    You use height and base measurements to find the areas of both figures.

    The triangle looks about half as big as the parallelogram, so take half of the parallelogram's area to find the triangle's area.

    If you divide the area of the parallelogram by 2, you get the area of the triangle.

    The area of the triangle is one-half the area of the parallelogram.

    Twice the area of the triangle equals the area of the parallelogram.

    Using what you observed, what is the formula for the area of a triangle? Write the area on chart paper: $A = \frac{bh}{2}$, or $\frac{1}{2}bh$.

14. You can verify this by showing the area of $\triangle EFG$ is the same as the area of $\triangle DEF$ and the sum of their areas is equal to the area of the parallelogram. First select points E, F, and G and choose **Construct | Triangle Interior.** Select $\triangle EFG$ and choose **Measure | Area.** Choose **Number | Calculate** and add the two triangle measurements. The sum will equal the area of the parallelogram.

15. You can also show that the triangle is half the area of the parallelogram by going to page "Double Triangle." Press *Double Triangle* to activate the animation.

16. ***How can you find the area of a triangle if you know the area of a parallelogram with the same base and height?*** You may wish to have students respond individually in writing to this prompt.

17. If time permits, discuss the Explore More. Students should note that the area remains constant because the height and the base do not change.

ANSWERS

15. The area of the parallelogram remains constant under shearing because the height and base of the parallelogram stay constant, and the area depends on only those two measures. Changing the base or the height of the parallelogram changes its area.

21. The area of the parallelogram equals $(m\overline{DF})(m\overline{EH})$.

22. $A = bh$

28. The area of the triangle is exactly half the area of the parallelogram.

$$A = \frac{bh}{2}$$

29. During the animation, the area of the parallelogram should not change because neither the height nor the base changes. Area remains constant under shearing for any figure because every cross section parallel to the shear line remains a constant length.