Parallel Pairs

For GSP5

Name:

In this activity you'll discover a relationship between the areas of parallelograms and triangles by investigating a process called *shearing.* This will help you write area formulas for parallelograms and triangles.

CONSTRUCT

1. In a new sketch, choose **Edit│Preferences│Text** and check **Show labels automatically: For all new points.** Click **OK.**
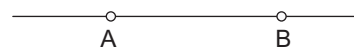
2. Construct a horizontal line, $\overleftrightarrow{AB}$.
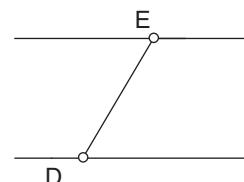
3. Construct point *C* above $\overleftrightarrow{AB}$.

4. Now you'll construct a line parallel to $\overleftrightarrow{AB}$ through point *C*.

   Select point *C* and $\overleftrightarrow{AB}$, and then choose **Construct│Parallel Line.**

Steps 2–4

5. Hide points *A, B,* and *C* by selecting the points and choosing **Display│Hide Points.**

6. Construct $\overline{DE}$ from the bottom line to the top line.
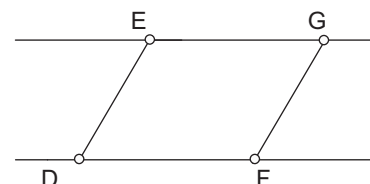
Steps 5 and 6

7. Construct point *F* on the bottom line.

8. Construct a line through point *F* parallel to $\overline{DE}$.

9. Construct point *G* where this line intersects the top line.

Steps 7–9

10. Construct interior *DEGF* by selecting the vertices in order and choosing **Construct│Quadrilateral Interior.**

11. Hide $\overleftrightarrow{FG}$.

12. Construct $\overline{FG}$.

EXPLORE

13. Measure the area of parallelogram *DEGF* by selecting the interior and choosing **Measure│Area.**

14. Observe the area measurement as you drag in each of these ways.

   • Drag point *E* to shear the parallelogram.

   • Drag point *D* or point *F* to change the base of the parallelogram.

   • Drag either $\overleftrightarrow{EG}$ or $\overleftrightarrow{DF}$ up or down to change the height.

15. Which of these actions change the area and which don't? Explain why you think this is so.

A height of a parallelogram is the shortest distance between two opposite bases. Now you'll construct an *altitude*—a segment that is perpendicular to the bases and whose length is a height of the parallelogram.

CONSTRUCT

16. Select point *E* and $\overleftrightarrow{DF}$, and then choose **Construct│Perpendicular Line.**
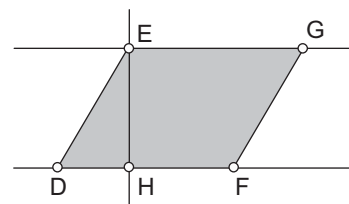
17. Construct point *H* at the intersection of $\overleftrightarrow{DF}$ and the perpendicular line.

18. Hide the perpendicular line.

19. Construct base $\overline{DF}$ and altitude $\overline{EH}$.

EXPLORE

20. Select $\overline{DF}$ and $\overline{EH}$ and choose **Measure│Length.**

21. You will use these measurements to calculate an expression equal to the area of the parallelogram.

   Choose **Number│Calculate** to open the Sketchpad Calculator.

   Click once on a measurement to enter it into a calculation.
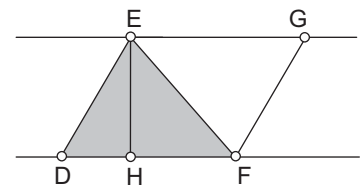
   Write down your expression.

22. Write a formula for the area of the parallelogram using *A* for area, *b* for base, and *h* for height.

Next, you'll investigate how the area of the parallelogram is related to the area of a triangle.

CONSTRUCT

23. Hide the interior of the parallelogram.

24. Construct diagonal $\overline{EF}$.

25. Construct triangle interior *DEF.*

EXPLORE

26. Measure the area of triangle *DEF.*

27. Drag point *E* and observe the area measurements.

28. How is the triangle's area related to the parallelogram's area? Write a formula for the area of the triangle using *A* for area, *b* for base, and *h* for height.

EXPLORE MORE

29. Make an action button to animate point *E* along its line. Explain what this animation demonstrates about shearing.

    Select point *E* and choose **Edit│Action Buttons│Animation.**