INTRODUCE

Project the sketch for viewing by the class. Expect to spend about 5 minutes.

1. Students should have already completed the activities Prism Nets and Prism Dissection. Show students the sample full-page net you cut out, and fold it to form a regular pentagonal pyramid. Explain, ***Today you'll use Sketchpad to find the surface area of regular pyramids. You'll start with a pentagonal pyramid and then figure out a formula that works for other regular pyramids.***
2. Open **Pyramid Dissection.gsp.** Explain, ***On page "Pyramid," you'll review how to change the viewing angle and how to change the dimensions and number of sides.*** If students are not already familiar with the controls, model the use of *spin, pitch,* and *roll,* and the use of *N*, *R*, and *L*.
3. ***To find the surface area of a pyramid, what would you have to measure?*** Some students might provide a general description that you must measure the areas of the base and the five triangular lateral faces. Others might give more detail, describing how to measure the base and height of each triangle. ***How can a net help you?*** Students should see that the area of the net is equal to the surface area of the pyramid.
4. Holding up the folded pyramid, ask, ***Where would you measure the height of this pyramid?*** Encourage students to notice that there are two possible height measurements: the vertical distance from the center of the base to the vertex (height of the pyramid) and the distance from the base of a lateral face to the vertex (height of a triangle). Encourage students to discuss how these two different measurements might be useful, and discuss with them why it's important to avoid confusion by distinguishing the two heights. You may want to ask them to propose their own names for these measurements. Explain, ***On your worksheet, the distance from the base of a triangular face to the vertex is called the* slant height *and is labeled l.***
5. Students should have previous experience with the Construct and Measure menus. You may want to briefly review how to construct a midpoint, how to measure the distance between two points, how to change the label of a measurement, and how to click on an object in the sketch to enter it into the Calculator.

DEVELOP

Expect students at computers to spend about 30 minutes.

6. Assign students to computers and tell them where to locate **Pyramid Dissection.gsp.** Distribute the worksheet. Tell them, ***First review the controls on page "Pyramid" and answer the questions in steps 3 and 4. Then you'll go to page "Base" and find the area of the base. Try to figure out the measurements and calculations you need without using the hint.***

7. Give students time to work on their measurements and calculations. If necessary, remind them to enter existing measurements into the Calculator by clicking on them rather than by typing in the numbers. Some students may be ready to go on to page "Faces" before others have finished their calculations. If they do so, you can check their results for worksheet steps 6, 7, and 9. For step 9, make sure they answered approximately 6.29 for the ratio of the perimeter to r, and approximately 3.14 for the ratio of the area to r^2. Also make sure they've written an explanation for why they got these values.

   If students have different answers for worksheet step 9, check to make sure that they used the value of n and the actual measurements rather than typing in numbers, so that their results are correct no matter how they change the number of sides.

8. When most students have finished worksheet step 9, call the class together and ask several students to report their measurements and calculations of perimeters and areas from worksheet steps 6 and 7. Ask, ***What values did you get for the two calculations in step 9?*** Encourage students to explain why the two ratios come out to 2π and π. You may want to ask them whether the values are exactly 2π and π, and if not, why not? How could they measure the percentage by which the ratios differ from 2π and π?

9. Ask students, ***When you did your calculations, why was it important to click on the measurements in the sketch rather than just typing in the number?*** Encourage students to observe that they want their calculations to be correct even when they change the pyramid dimensions or the number of sides.

10. Tell students, ***Now you'll calculate the area of the lateral faces of the pyramid. Go on to page "Faces" and then to page "Area" when you're ready. If you finish early, try some of the Explore More problems.***

11. Give students time to work on worksheet steps 10–13. In step 12, some students may need a hint; you can suggest that they see what happens if they divide their numeric answer by *rl*.

    Consider assigning some or all of the Explore More questions, depending on your curriculum needs. If you do so, allot additional time.

SUMMARIZE

Project the sketch. Expect to spend about 10 minutes.

12. Have students discuss their results. Here are some questions for discussion.

    How did you find the area of the base?

    How did the net help you to understand the problem of finding the surface area?

    You ended up with a formula that involved the values of n, r, l, and the length of a side of the base. Do you really need all of these values?

    Encourage students to realize that these are interdependent. At the middle school level, you might discuss that for a given number of sides, the side length increases in proportion to the value of *r*. As you drag *R*, observe the side length and note that the triangles used to find the area of the base remain similar. At the high school level, you can use trigonometry to relate the side length explicitly to the values of *r* and *n*.

13. ***What interesting things happen as n increases?*** Students will observe that the shape becomes more and more like a cone. ***What happens to the area of the base?*** Students will have noticed that the ratio of area to r^2 in worksheet step 9 approaches π. They should be ready to draw the conclusion that the area of the polygon approaches πr^2, the area of a circle. ***What happens to the total area of the lateral faces?*** Some students will have noticed that the ratio of the perimeter to *r* in worksheet step 9 approaches 2π. Encourage students to explain how they could use the perimeter to make it easy to calculate the sum of the areas of these faces, by calculating *perimeter* $\times$ *slant height*/2. (Explore More worksheet step 15 explicitly asks students to use these facts to develop a formula for the surface area of a cone.)

EXTEND

What other questions might you ask about pyramids? Encourage all inquiry. Here are some ideas students might suggest.

What do pitch, roll, and spin really do? Are there other similar movements?

Why are the circumference and the area of a circle given by the familiar formulas? Is this activity a proof?

Can you do something like this to find the volume of the pyramid or cone?

Can you get the surface area of other curved figures, like a sphere, by taking some figure and increasing the number of faces?

ANSWERS

3. Student answers will vary. The *pitch* control allows you to see a top view, so that the pyramid looks like a regular polygon. In this view the *spin* control rotates the polygon about its center.

4. You cannot make the slant height l smaller than the value of r because the slant height must reach at least from the edge of the base to the center. When the slant height is equal to r, the pyramid is completely flat, with a height of 0. When the slant height is large compared to the radius, the pyramid is tall and skinny.

6. If students use s for the length of one side of the base, the perimeter is ns. Numeric results will vary and should change as students manipulate the dimensions.

7. One way to measure the area of the base is to think of it consisting of n triangles (as suggested by the hint), to measure the base (s) and height (r) of one triangle, and then use $A = sr/2$ to find its area. To find the area of the entire base, students can multiply by n, with the result that the total area is given by $nsr/2$.

9. The ratio of the perimeter to r is approximately 6.29—nearly 2π—and the ratio of the area to r^2 is approximately 3.14—nearly π. When $n = 60$, these values are accurate to about two decimal places.

11. The area of one lateral face is $sl/2$, and the sum of the areas of all these faces is $nsl/2$. If students use the perimeter p in their calculations, they will use the formula $pl/2$. Numeric answers will vary.

12. When the number of sides is large ($n \geq 50$), the perimeter approaches $2\pi r$, so the area of the sides approaches πrl.

13. The total area of the pyramid is the sum of the area of the base and the lateral faces: $nsr/2 + nsl/2$. Students may factor this to write it as $ns(r + l)/2$. Numeric results will vary.

15. When the value of n is large, the base becomes very nearly a circle, and the base area can be written as πr^2. The sum of the areas of the lateral faces approaches πrl, so the surface area of a cone is given by $\pi r^2 + \pi rl$, or $\pi r(r + l)$.

16. The height h of the pyramid (measured from the center of the base to the vertex) and the distance r form two legs of a right triangle, with the slant height l forming the hypotenuse. By the Pythagorean Theorem, $r^2 + h^2 = l^2$. If $l = 15$ cm and $r = 9$ cm, $h = 12$ cm. If $h = 5$ cm and $r = 12$ cm, $l = 13$ cm.