SKETCH AND INVESTIGATE

Q1 The quadrilateral whose sides connect the midpoints of any quadrilateral is a parallelogram. The measurements support this conjecture because they show that the opposite sides of the midpoint quadrilateral are equal in length and that opposite sides have equal slope (and therefore are parallel).

Q2 A diagonal divides the quadrilateral into two triangles. Two sides of the midpoint quadrilateral are midsegments of these triangles. This means they are both parallel to the diagonal and half as long. If one pair of opposite sides of a quadrilateral are both equal in length and parallel, the quadrilateral is a parallelogram. (Students might construct the other diagonal and use a second pair of triangles to show that the other pair of sides of the midpoint quadrilateral are also equal in length and parallel.)

EXPLORE MORE

9. A midpoint quadrilateral of a midpoint quadrilateral is still just a parallelogram. Successive midpoint quadrilaterals are alternately similar; that is, the third midpoint quadrilateral is a parallelogram similar to the first, the fourth is similar to the second, and so on. These parallelograms converge on the point of intersection of segments connecting midpoints of opposite sides.

10. The area of the midpoint quadrilateral is half the area of the original quadrilateral. (As an extra challenge, ask students to prove this is true.)

11. The conditions under which a midpoint quadrilateral is a special parallelogram are not obvious. In general, the midpoint quadrilateral of a trapezoid is a parallelogram, that of an isosceles trapezoid is a rectangle, that of a parallelogram is a parallelogram, that of a kite is a rectangle, that of a rhombus is a rectangle, that of a rectangle is a rhombus, and that of a square is a square. See the activity Special Midpoint Quadrilaterals for more discussion.

12. The page "Special Midpoint Quads" of **Midpoint Quadrilaterals Work.gsp** illustrates the most general quadrilaterals whose midpoint quadrilaterals are special parallelograms. This question is the subject of the activity Special Midpoint Quadrilaterals. The midpoint quadrilateral of any quadrilateral whose diagonals are equal is a rhombus. The midpoint quadrilateral of any quadrilateral whose diagonals are perpendicular is a rectangle. The midpoint quadrilateral of any quadrilateral whose diagonals are equal and perpendicular is a square.