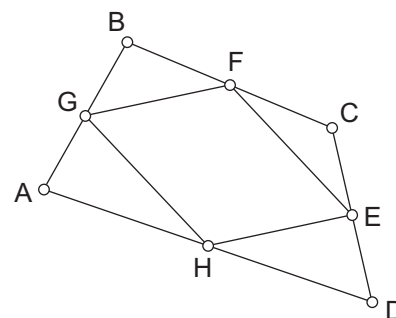# Midpoint Quadrilaterals

For GSP5

In this investigation you'll discover something surprising about the quadrilateral formed by connecting the midpoints of another quadrilateral.

SKETCH AND INVESTIGATE

If you select all four sides, you can construct all four midpoints at once.

1. Construct quadrilateral *ABCD*.

2. Construct the midpoints of the sides.

3. Connect the midpoints to construct another quadrilateral, *EFGH*.

4. Drag vertices of your original quadrilateral and observe the midpoint quadrilateral.

5. Measure the four side lengths of this midpoint quadrilateral.

Q1 Measure the slopes of the four sides of the midpoint quadrilateral. What kind of quadrilateral does the midpoint quadrilateral appear to be? How do the measurements support that conjecture?

6. Construct a diagonal.

7. Measure the length and slope of the diagonal.

8. Drag vertices of the original quadrilateral and observe how the length and slope of the diagonal are related to the lengths and slopes of the sides of the midpoint quadrilateral.
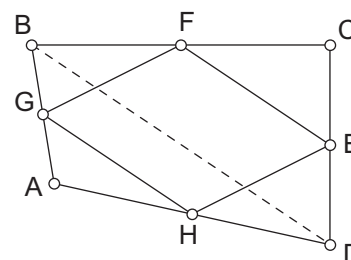
Q2 The diagonal divides the original quadrilateral into two triangles. Each triangle has as a midsegment one of the sides of the midpoint quadrilateral. Use this fact and what you know about the slope and length of the diagonal to write a paragraph explaining why the conjecture you made in Q1 is true.

EXPLORE MORE

9. Construct the midpoint quadrilateral of the midpoint quadrilateral. Then construct *its* midpoint quadrilateral. Do this two or three more times. Describe any patterns you see in the midpoint quadrilaterals.

10. Construct the polygon interiors of a quadrilateral and its midpoint quadrilateral. Measure their areas. Make a conjecture about these areas.

11. What's the midpoint quadrilateral of a trapezoid? An isosceles trapezoid? A parallelogram? A kite? A rhombus? A rectangle? A square? Organize and explain your findings.

12. Under what conditions is a midpoint quadrilateral a rectangle? A rhombus? A square? See if you can construct the most general quadrilateral whose midpoint quadrilateral is one of these.